



# Construction and systematic assessment of relative deformation histories

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## Abstract

The systematics of deformation histories have been investigated using basic combinatorial mathematics. Using this approach, the number of deformation histories and the number of relationships associated with a given number of structures can be calculated. The range of possible deformation histories can also be determined. Using this information, the ambiguous nature of all deformation histories is examined together with the impact of unobserved relationships. The number of possible deformation histories that are compatible with a collection of age relationships is used as a crude measure of confidence in those histories. A procedure is described for the construction of deformation histories that is systematic, repeatable and rapid and within which ambiguities can be easily recognised. © 1999 Elsevier Science Ltd. All rights reserved.

## 1. Introduction

After approximately 120 years of research in structural geology and approximately 40 years of research underpinned by continuum mechanics it would appear unnecessary to consider such a fundamental part of structural analysis as deformation histories. However, we contend that some of the systematics of establishing deformation histories have not been fully explored and, the impact of unobserved relationships on the confidence levels that may be placed on individual deformation histories has not been addressed. This has led to the publication of non-unique deformation histories where valid alternatives have not been recognised. Also, many of the great debates concerning the validity of different deformation histories for a particular area have arisen from unobserved relationships. A lack of appreciation of the ambiguities generated by unobserved relationships has led to cases where compatible, but apparently different deformation histories have been proposed, and the compatibility has not

been recognised by the protagonists. In other cases, the absence of even a semi-quantitative estimate of confidence has led to lengthy arguments over the relative merits of two deformation histories which may, in reality, have either equal or widely different confidence levels associated with them. Using simple combinatorial mathematics we provide a framework within which the deformation history of an area can be established as objectively as possible. Based on this framework we identify procedures by which this process is repeatable and provides a semi-quantitative estimate of confidence. An essential element of these procedures is the easy recognition of ambiguities.

To understand the systematics of deformation histories a distinction must be made between cyclic deformation histories (e.g. Carreras et al., 1977; Evans and White, 1984; Mawer and Williams, 1991) and non-cyclic or linear deformation histories (e.g. Soper, 1971; Soper and Brown, 1971; Soper and Wilkinson, 1975). A cyclic history consists of a collection of structures that deform and are deformed by other structures in a cycle that is repeated many times. Non-cyclic histories are characterised by structures that are the products of a series of sequential events. These events may create one or more structures. The various combinations and

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permutations of structures that form parts of cyclic and non-cyclic deformation histories are examined in this paper. By considering all possible sequences and arrangements of structures the systematics of deformation histories can be established. The application of some of these concepts is illustrated using a published example (Soper, 1971; Soper and Brown, 1971; Soper and Wilkinson, 1975; Evans and White, 1984).

## 2. Systematics of deformation histories

Given the importance of deformation histories in structural analysis it is surprising how little is written on deformation histories and their construction. This statement applies equally well to individual texts and the number of texts (both books and papers). For example, in terms of length and depth, the description given by Hobbs et al. (1976), pp. 368–369 (no criticism intended) is typical of many [e.g. Zwart, 1960, 1963; Spry, 1969; Vernon, 1976; Hancock, 1985; Barker, 1990; Price and Cosgrove, 1990; Passchier and Trouw, 1996 (no criticism intended)]. “During the process of mapping it is necessary, in multiply deformed areas, to build up a picture of the interrelationship of the various structures. By this means the sequence of events is determined.... The various structures are observed and wherever possible ascribed to style groups. All overprinting relationships are recorded and the style groups ascribed to fold generations, wherever possible”. Exceptions are Turner and Weiss (1963), Park (1969), Williams (1970, 1985) and Tobisch and Fiske (1982). There are numerous structural texts omitted from the list above (e.g. Ramsay, 1967; Park, 1983; Ramsay and Huber, 1983, 1987; Ragan, 1985; Hatcher, 1990; Twiss and Moores, 1992; Davis and Reynolds, 1996). In general, these provide reasoned accounts of how the relative ages of individual pairs of structures can be established (e.g. superposed folds, Ramsay, 1967) but they do not give any formal descriptions of the ways in which deformation histories are constructed.

Park (1969) highlighted the problems of correlating structures and overprinting relationships from one area to another. He evaluated the use of style, orientation and foliation and several authors have developed some of these ideas further (e.g. Williams, 1970, style; Tobisch and Fiske, 1982, orientation; and Williams, 1985, foliation). Although, we share most of the concerns expressed in these papers, for the purposes of this contribution, we will assume that overprinting relationships can be established unambiguously and correlations can be undertaken with a reasonable degree of confidence. These assumptions enable us to address separate, but equally im-

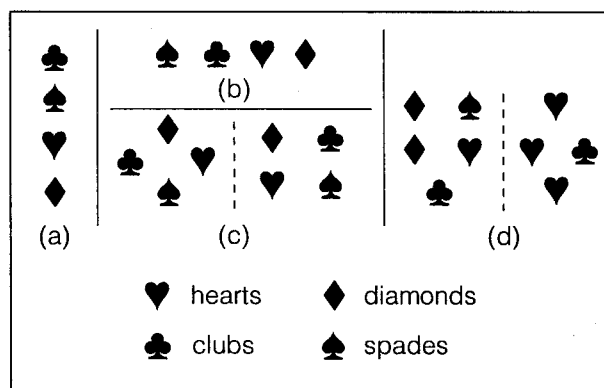


Fig. 1. Various non-cyclic deformation histories. Each symbol represents an individual structure or a set of identical structures. The structures are arranged such that the oldest structures are at the bottom of the diagram and the youngest at the top. The symbols for synchronous structures are placed next to each other. (a) A serial, non-cyclic deformation history in which each of the four structures appears only once and they are progressively younger in age. (b) A parallel, non-cyclic deformation history in which each of the four structures appears only once and all of the structures are synchronous. (c) Two non-cyclic deformation histories in which each of the four structures appears only once. In the first deformation history one pair of structures are of the same age and in the second two pairs of structures are synchronous. (d) Two different non-cyclic deformation histories in which one of the structures appears more than once ('diamonds' in the first case and 'hearts' in the second).

portant problems associated with the construction of deformation histories.

In the figures, symbols are used to represent different sets of structures. These may be foliations, folds, porphyroblasts, fractures or whatever! The symbols were chosen because, unlike alpha-numeric characters, they have no generally accepted sequence. An individual symbol may represent either, an individual structure that might be observed in a thin section or encountered in an exposure or, a set of identical structures with identical age relationships that may be found throughout a study area. We do not draw a distinction between individual structures and sets of structures since the issues addressed are common to both cases. Each line of one or more symbols in Figs. 1, 2, 4(a), 5 and 6 represent different stages in a deformation history. A stage is characterised by a unique association of one (Fig. 1a) or more (Fig. 1b and c) structures. A structure may appear in more than one successive stage (Fig. 1d). This situation may arise when either, identical structures occur at slightly different times (e.g. intrusive sheets) or, when the growth of one structure overlaps with the growth of another (e.g. porphyroblasts that are synchronous with and post-date a foliation).

Many of the deformation histories include two or more structures that are synchronous. For porphyroblasts and foliations there are well established geometrical relationships that indicate whether they are

Table 1  
Systematics of non-cyclic deformation histories in which a structure appears only once

Number of structures	Number of relationships	Number of histories	Apparent minimum number of relationships
1	0	1	0
2	1	3	1
3	3	13	2
4	6	75	3
5	10	631	4

synchronous or not (although the reliability of these relationships has been questioned, e.g. Johnson and Vernon, 1995 and references therein). For meso-scale structures it is less clear what criteria may be used. Mutual overprinting relationships observed at different locations have been successfully applied to fracture systems (Horsfield, 1980; Grocott, 1981; Hancock, 1985). However, as will be shown later, the absence of evidence that two structures are of different age, combined with identical relationships to older and younger structures, cannot be used with confidence.

As a starting point we will consider non-cyclic deformation histories in which identical structures appear only once (e.g. Fig. 1a–c). For a collection of different structures the number of relationships that must be established is given by

$$p_n = \frac{n(n-1)}{2} \quad (1)$$

where,  $n$  = the number of different structures and  $p_n$  = the number of relationships for a non-cyclic deformation history. Note that, in this system, the relation-

ships of one type of structure to *all* of the others must be established. The numbers of possible deformation histories are given in Table 1 and examples for one, two and three sets of structures are shown in Fig. 2. Thus, for a given number of different structures the number of relationships and number of possible deformation histories can be determined and these values will be used as the basis for further calculations.

Many authors have interpreted certain collections of different structures as the products of cyclic deformation histories. Considerable care must be taken with the term cyclic deformation. We, together with others

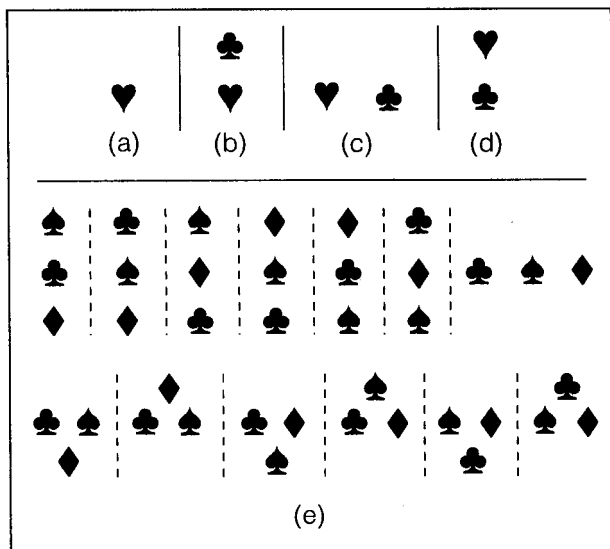


Fig. 2. Non-cyclic deformation histories for different numbers of structures. (a) One structure. (b)–(d) The three possible deformation histories for two structures. (e) The 13 possible deformation histories for three structures.

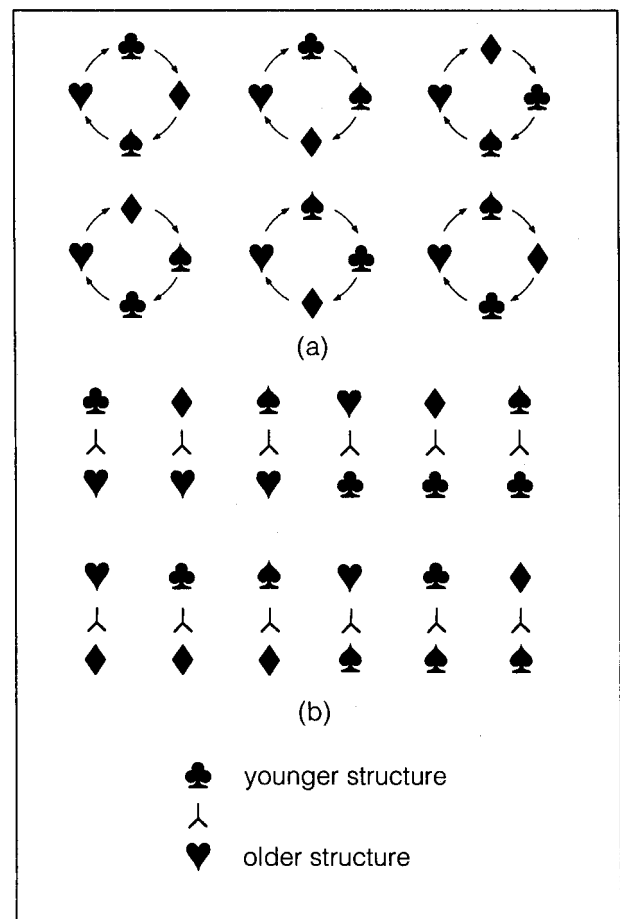


Fig. 3. (a) The six possible cyclic deformation histories for four structures. (b) The 12 relationships which substantiate the cyclic deformation histories in part (a).

Table 2  
Systematics of cyclic deformation histories in which a structure appears only once

Number of structures	Number of relationships	Number of histories	Apparent minimum number of relationships
1	—	—	—
2	2	1	2
3	6	2	3
4	12	6	4
5	20	24	5

(e.g. Mawer and Williams, 1991), follow the usage in chemistry and biochemistry where, the products of one step in the cycle are used in later steps. In some rock deformation experiments the term cyclic loading is used when perhaps repeated loading might be more appropriate. Because there are no ends to the cycle the number of histories is given by

$$h_c = (n - 1)! \tag{2}$$

(after Liu, 1968) where,  $h_c$ =the number of cyclic deformation histories in which a structure appears only once. For example, Fig. 3(a) displays the six cycles generated by four structures. Each type of structure must overprint every other type of structure thus, the number of relationships is given by

$$p_c = n(n - 1) \tag{3}$$

where  $p_c$ =number of relationships for a cyclic deformation history (Table 2). Furthermore, all six cycles (Fig. 3a) require exactly the same pieces of evidence (Fig. 3b) to substantiate them. Thus, from overprinting relationships it is impossible to determine the sequence

of structures in a cyclic deformation history. If less than  $n(n - 1)$  relationships are observed, the sequence indicated by the relationships is simply related to the missing evidence. There is a one in  $h_c$  chance that the sequence is real but there is no way of testing whether it is.

The sequence of events in a cyclic deformation can only be suggested through the application of a kinematic model (e.g. Mawer and Williams, 1991) but care must be taken to avoid circular arguments. If a specific kinematic model is used to constrain all or parts of a cyclic deformation history then that history *cannot* be presented as evidence for the operation of the kinematic model.

The set of relationships (Fig. 4b) which support the non-cyclic history shown in Fig. 4(a) are a subset of those (Fig. 4d) which substantiate the cyclic deformation history (Fig. 4c). These systematics combined with the real possibility of unobserved relationships creates substantial problems, but the following rules can be used. If greater than  $n(n - 1)/2$  relationships are observed the deformation history must be cyclic. Note that, this rule is independent of which relation-

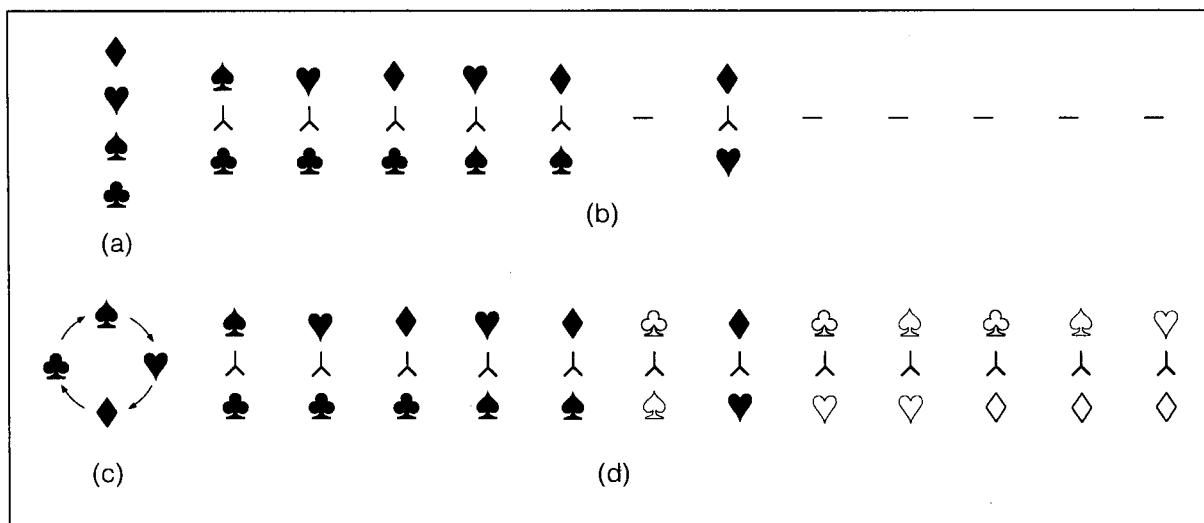


Fig. 4. (a) A serial, non-cyclic deformation history comprising four different structures. (b) The six relationships which support the deformation history shown in part (a). Note that the other possible non-cyclic histories would be supported by six different pieces of evidence. (c) One of the six possible cyclic deformation histories comprising four structures. (d) The 12 relationships which substantiate the cyclic deformation history shown in part (c). The solid symbols indicate relationships common to both histories shown in parts (a) and (c). The open symbols indicate relationships that substantiate the cyclic deformation history but do not support the particular non-cyclic deformation history shown in part (a).

ships are seen. If  $n(n-1)/2$  relationships are observed and they are those and only those common to both histories then, the deformation *could* be non-cyclic though, it may well be cyclic. If less than  $n(n-1)/2$  relationships are observed the deformation history may be cyclic or non-cyclic. Thus, under certain circumstances overprinting relationships can be used to demonstrate that a deformation history is cyclic or either non-cyclic or cyclic but they cannot be used to determine the sequence.

Using overprinting relationships, Soper (1971), Soper and Brown (1971) and Soper and Wilkinson (1975) established a four-stage, non-cyclic deformation history ( $D_1$ – $D_4$ ) for mylonitic rocks exposed at the northern end of the Moine thrust belt of northwest Scotland. During  $D_1$  the main mylonitic foliation ( $S_1$ ) was formed together with intrafolial folds of diverse orientations ( $F_1$ ). Occasionally the  $S_1$  foliation is axial planar to the  $F_1$  folds. The mylonitic foliation continued to develop during  $D_2$  and was associated with the growth of a grain shape/mineral lineation ( $L_2$ ) and small-scale folds ( $F_2$ ). The axes of the  $F_2$  folds lie at small angles to the ESE-plunging  $L_2$  lineation. Locally developed  $F_3$  folds with N–S- or NNE–SSW-trending fold axes and easterly dipping axial planar crenulation foliations characterise the third phase of deformation.  $D_4$  comprises single and conjugate sets of kink bands. Key pieces of evidence that substantiate this history include: folding of the mylonitic foliation and colour banding by the various fold phases; deformation of post- $S_1$  and pre- $L_2$  veins; and patterns of superimposed folding. Evans and White (1984) did not draw a distinction between  $D_1$  and  $D_2$  and, following a model developed by Carreras et al. (1977) for small-scale shear zones, they interpreted the structures of  $D_1$  and  $D_2$  to be the products of a single continuous cyclic deformation. In their model the folds nucleated episodically and their axes rotated towards the ESE lineation with increasing strain as the foliation and lineation developed. Thus, the different stages of Soper and his co-workers represent different states of strain. Evans and White (1984) did not dispute any of the relationships reported by Soper and his co-workers nor did they introduce any new ones. Thus, both groups of workers used the same set of relationships to substantiate their particular deformation history. It was demonstrated above and in Fig. 4 that the existence of cyclic deformation histories can be established from the number of relationships and the number of structures that have been observed. Sadly insufficient information was provided in order to complete this calculation. However, since both groups provide the same evidence then, it is impossible to state whether the history is cyclic or non-cyclic and both possibilities must be considered equally valid. Implicitly Evans and White prefer their own model and conclude (p. 371, Evans and

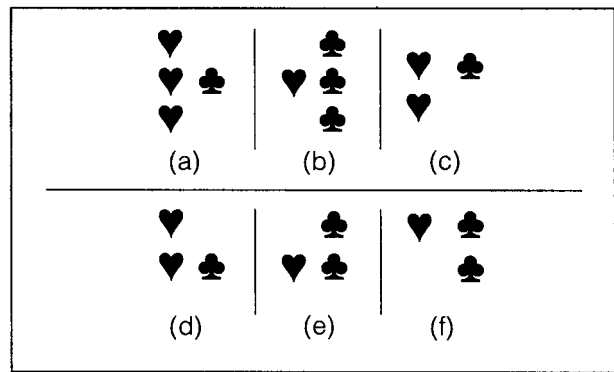


Fig. 5. Six non-cyclic deformation histories for two structures within which one of the structures appears more than once. The repeated symbol and the deformation history that contains them represent two different geological cases: (1) where identical structures occur at different times, e.g. intrusive sheets of slightly different age but the same orientation; and (2) when the growth of one structure overlaps with the growth of another, e.g. porphyroblasts that have grown syn- and post-kinematically with respect to a foliation.

White, 1984) that “the tectonic framework of Soper and his co-workers is in need of revision” and, later that the  $D_1$ – $D_4$  scheme is too restrictive. Since it cannot be established (from the published data) whether or not the deformation was cyclic then one model cannot be preferred over the other and it is premature to dismiss the non-cyclic history. Indeed, Evans and White (1984) provide no evidence that refutes the history proposed by Soper and his co-workers. The validity of other published deformation histories can be assessed in a similar manner.

Similar calculations can be performed for non-cyclic deformation histories where a structure appears more than once in consecutive stages (Fig. 1d). In such deformation histories some of the structures must be synchronous. For the moment we consider only those cases where synchronicity can be established unequivocally using geometrical criteria (e.g. syn-kinematic porphyroblasts) rather than mutual overprinting relationships. The numbers of deformation histories are relatively large; six for two structures (Fig. 5) and 51 for three. The patterns of relationships are complex and share similarities with both non-cyclic and cyclic deformation histories. For the sake of brevity and clarity the systematics of these deformation histories will not be considered in any detail but they must be considered briefly in order to illustrate an important principle. Since it is impossible to know whether or not a piece of evidence is missing, there is an element of doubt associated with every deformation history as to its validity. This problem will be illustrated using cases with two structures. The relationship which supports the deformation history shown in Fig. 2(b) is a subset of those which substantiate the deformation histories shown in Fig. 5(a–c and e). It is also a subset of

the two relationships that indicate the existence of a cyclic deformation history (not shown). Therefore, even if the history shown in Fig. 2(b) is the true deformation history it *could* be part of five others and a critical relationship *may* exist that has not been observed. Thus, it is one of six possible histories that are all compatible with the data (Fig. 2b). This problem is not unique to the deformation history shown in Fig. 2(b). The deformation histories shown in Fig. 2(c and d) could be parts of seven and six histories, respectively. These calculations convey some indication of the levels of confidence that may be placed on individual deformation histories. However, we have not formalised them as probabilities. To do so it must be assumed that each deformation history is equally likely to occur and, although there is no way of knowing, this seems improbable. This problem does not end there. The deformation histories in Fig. 5(c and e) share the same relationships and these are a subset of those which substantiate the deformation histories in Fig. 5(a and b). Thus, even if all of the relationships are observed, it is impossible to distinguish between the deformation histories displayed in Fig. 5(c and e) and it is impossible to preclude the possibility that either one of these histories is one of those displayed in Fig. 5(a and b) and that a relationship has not been observed. Therefore, the two relationships 'hearts' and 'clubs' are synchronous and 'clubs' are younger than 'hearts', and are compatible with four deformation histories (Fig. 5a–c and e). These statements are also true of the deformation histories displayed in Fig. 5(a, b, d and f). Similarly, the deformation histories shown in Fig. 5(a and b) share the same relationships and either history could be the correct one. The relationships associated with a cyclic deformation history form a subset of those which support the non-cyclic histories displayed in Fig. 5(a and b). Hence, even when both pieces of evidence are seen, the cyclic deformation history could still be a part of a non-cyclic history (note that, in this instance, the number of relationships are no help). Once the ambiguities are recognised it is a relatively simple task to determine how many deformation histories a collection of relationships are compatible with.

These calculations can be undertaken for larger numbers of structures, but they become more involved due to the large numbers of possible deformation histories. These ambiguities are inherent in the patterns of combinations and permutations associated with a given number of structures. It must be emphasised that they arise from relationships that *could* be present rather than relationships that *are* present and have not been observed. Thus, there is nothing that can be done to resolve them since this would involve searching for relationships that can never be found! In these circumstances, the most productive approach is to accept that

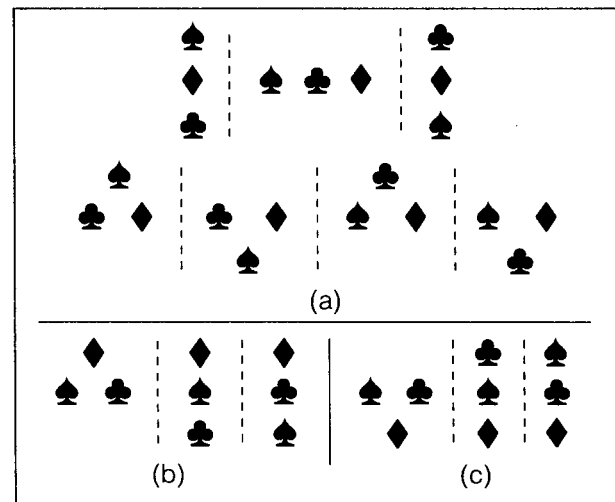


Fig. 6. The 13 possible non-cyclic deformation histories for three structures. (a) Even when the relationships between 'spades' and 'clubs' is unknown these seven can be recognised as the true deformation histories. (b) When the relationship between 'spades' and 'clubs' is unknown and 'diamonds' are consistently younger than other structures the deformation history must be one of these three possible histories. (c) When the relationship between 'spades' and 'clubs' is unknown and 'diamonds' are consistently older than other structures the deformation history must be one of these three possible histories.

these ambiguities exist and to pay them due attention during the construction of deformation histories. When two published deformation histories are compared it is worth considering whether or not they share the same relationships and, therefore, might be equally valid.

Note that, if mutual overprinting is used as evidence that the two structures are synchronous then it is impossible to distinguish between the various deformation histories in Fig. 5. Indeed, only one history can be inferred (that displayed in Fig. 2c) which may be erroneous.

### 3. Impact of unobserved relationships

In this section the impact of unobserved relationships will be considered. This is a separate and distinct problem from that described in the previous section. In this case relationships exist but they are, for whatever reason, not observed.

Inspection of Fig. 2(e) suggests that each of the deformation histories could be constrained by just two relationships and that the minimum number of relationships is given by

$$m_n = n - 1 \quad (4)$$

where  $m_n$  = the apparent minimum number of relationships for a non-cyclic deformation history in which a

structure appears only once. However, this logic is flawed and closer inspection indicates that only certain relationships, not just any two, will correctly constrain the deformation history. For example, in Fig. 2(e) the observations that 'spades' and 'clubs' are younger than 'diamonds' are compatible with three deformation histories whereas, the observations that 'spades' are younger than 'clubs' and 'clubs' are younger than 'diamonds' are compatible with only one history. Since the true deformation history is unknown, it is impossible to judge whether or not the correct relationships have been observed. When and only when all of the different types of structures are synchronous will  $n - 1$  relationships be sufficient. Thus, for non-cyclic deformation histories it is important to consider the effects of unobserved relationships. This problem does not extend to cyclic deformation histories since overprinting relationships cannot be used to constrain the sequence of structures.

From Table 1 we know that to constrain a simple non-cyclic deformation history associated with, for example, three structures we must observe three relationships and there are 13 possible deformation histories. If *one* of the three relationships is not seen then only seven of the 13 histories can be recognised unequivocally (Fig. 6a). The remaining histories form two groups of three (Fig. 6b and c) and while it is possible to distinguish one group ('diamonds' are the youngest) from the other ('diamonds' are the oldest), it is impossible to determine which of the deformation histories within a group is the correct one. Thus, with only one piece of the data missing a significant proportion of the possible deformation histories cannot be distinguished. One might argue that one piece of evidence out of three is a large proportion of the data and it is having a disproportionately large effect. Preliminary calculations indicate that, for large numbers of structures, although the proportion of missing data is smaller (say one piece out of six) the larger number of possible deformation histories (Table 1) ensures a significant effect.

It is pleasing that, even with unobserved relationships, some deformation histories can be shown to be correct. However, it remains a matter of some concern that significant numbers of deformation histories cannot be distinguished and that many possible histories have low probabilities of being correct.

For a given number of different structures, the possible deformation histories can be determined (Figs. 2 and 3). From Eqs. (1) and (3), the number of relationships needed to substantiate a history can be calculated. Therefore, the amount of missing evidence can be assessed. A proposed history with some unobserved relationships should be compared with other possible histories. Other valid histories will share the observed relationships and all possible histories should be

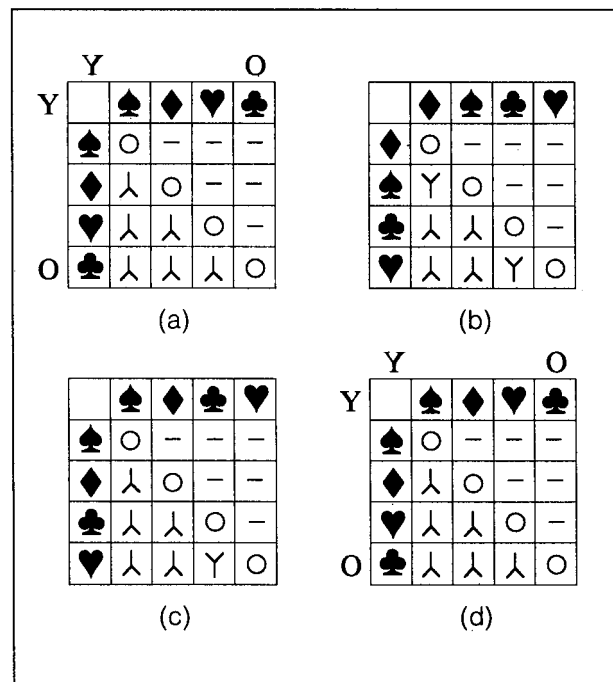


Fig. 7. Younging tables for deformation histories with four structures. O=oldest structure, Y=youngest structure, open circle=no relationship. The younging symbol is used in the usual way (see Fig. 3). The table should be read thus: the structure in the column heading is younger than the structure in the row heading, e.g. 'diamonds' are younger than 'clubs'. (a) Pattern of relative ages for structures displayed in the correct order. The deformation history is non-cyclic and different types of structures appear only once. (b) Pattern of relative ages obtained from the deformation history displayed in parts (a). The different structures are written in any order. Note that, the second and fourth rows contain structures that are in the wrong position. (c) The same relationships plotted after the positions of 'diamonds' and 'spades' have been exchanged. Note that only row one contains a structure in the wrong position. (d) The same relationships plotted after the positions of 'clubs' and 'hearts' have been exchanged. Note that, the table now has the ideal form displayed in part (a) and the structures are in the correct order.

reported. The number of possible histories will convey an indication of the confidence that may be placed on them. When two published deformation histories are compared, their compatibility (given the amount and nature of any missing evidence) should be assessed first. If they are compatible (e.g. those histories shown in Fig. 6b are compatible) then, they should be considered to be equally valid. If they are not compatible the number of observed relationships as compared to the number of unobserved relationships can be used as a crude measure of the confidence which should be placed on each history.

The absence of evidence that two structures are of a different age, combined with identical age relationships to older and younger structures, is ambiguous and cannot be used to infer that the structures are synchronous.

#### 4. Construction of a deformation history

The following procedure was prepared using the systematics of deformation histories described above. It is aimed primarily at non-cyclic and cyclic deformation histories in which structures appear only once.

*Step 1.* As structures are identified calculate the number of relationships which can be expected [Eqs. (1) and (3)]. Revise this calculation as new structures are observed.

*Step 2.* Use (as appropriate) overprinting, cross-cutting and abutting evidence combined with correlation to establish the relative age relationships between the various structures. Record which relationships have been observed. From Step 1 and diagrams such as Figs. 4 and 6 determine which possible relationships have not been observed for both non-cyclic and cyclic deformation histories.

*Step 3.* Using knowledge gained during mapping (e.g. the geographical distributions of structures) seek out the unobserved relationships which may be present. Do this proactively rather than hoping for a relationship to be encountered. Use the calculations in Step 2 as a guide.

*Step 4.* When a significant number of relationships have been observed, test using the criterion  $p > p_n$  where  $p$  is the number of observed relationships, whether or not the deformation history is cyclic. For  $p \leq p_n$  the deformation may be either non-cyclic or cyclic. For  $p > p_n$  the history must be cyclic (ignoring for the present non-cyclic deformation histories in which one structure appears more than once in successive stages).

*Step 5.* Record all of the relationships in a younging table (Fig. 7). Ideally one would like to prepare a younging table which is correct at the first attempt. In such a table the different structures appear in the correct sequence with the oldest structure at the bottom of the rows and on the right of the columns and the youngest of the structures at the top of the rows and on the left of the columns (Fig. 7a). When the structures are in the correct sequence all of the younging arrows point in the same direction (Fig. 7a). If the structures are placed in the table in the wrong order (e.g. Fig. 7b) then, preserving the relationships observed in the field, the arrows point in opposite directions (Fig. 7b).

*Step 6.* For rows which contain arrows pointing in the wrong direction move the header into the next row (above) by exchanging its position with the header above (Fig. 7c). Repeat the operation with the columns. Revise the table so that it honours both the field data and the new sequence (Fig. 7c). In Fig. 7 exchanging the headers 'diamonds' and 'spades' (Fig. 7b and c) has reversed the direction of

one of the younging symbols. Exchanging the position of the headers 'hearts' and 'clubs' reverses the last anomalous younging direction ((Fig. 7c and d). The revised table (Fig. 7d) has the ideal form displayed in Fig. 7(a), the sequence of structures is correct and the deformation history can be read from the table. Thus, if the headers of any rows containing anomalous younging directions are moved up the table (and the changes are also made to the columns) the construction of a deformation history from a collection of relationships can be undertaken efficiently and consistently.

Cyclic histories will use both the halves of the table but no ordering will be necessary because the sequence of structures cannot be determined from overprinting relationships.

For non-cyclic deformation histories in which a structure appears only once and, some of these structures are synchronous, younging tables can still be used to determine the sequence of structures. For such cases the synchronous structures are placed under one header and treated as a single entity in any manipulations. The tables are refined using the procedure outlined above with the proviso that any set of structures that are synchronous with two or more structures of different ages must appear in adjacent rows/columns.

*Step 7.* If a deformation history appears complete, i.e. a sequence of structures has been established which is supported by  $p_n$  relationships then, the history should be examined to determine whether or not it forms part of a larger, more complex non-cyclic history in which one or more structures appear more than once in successive stages (Figs. 1d and 5) or, a cyclic history (Fig. 3). Note that, more than  $n(n-1)/2$  relationships indicate that the deformation history is one of the last two types. Record the number of deformation histories which the proposed history may form a part. If  $p < p_n$  proceed to the next step, if not, draw up the deformation history including all possible histories. The number of possible histories can be taken as an indication of the confidence that may be placed on them.

*Step 8.* Using the number of observed relationships and the numbers of expected relationships [Eqs. (1) and (3)] assess the impact of unobserved relationships. Determine, for the observed relationships, all possible deformation histories. Younging tables can be used to recognise the ambiguities with relative ease. Unobserved relationships should be recorded by means of a question mark (?). The table should be refined so that all younging arrows point towards the top of the diagram (Fig. 7a). Alternative solutions can be found by trial and error by moving headers associated with unobserved relationships. Record and present all possible deformation his-



tories. The number of possible relationships can be taken as an indication of the confidence that can be placed on them.

## 5. Conclusions

Combinatorial mathematics (e.g. Liu, 1968) provides a useful framework within which deformation histories can be investigated. Using this approach, for a given number of different structures, the number of possible deformation histories and the number of relationships necessary to substantiate them can be calculated. Also, with a little effort, the various deformation histories can be determined.

The sequence of structures in cyclic deformation histories cannot be determined from overprinting relationships. Non-cyclic deformation histories, in which structures appear only once, always form parts of possible cyclic deformation histories and non-cyclic deformation histories in which one or more structures appear more than once in successive stages. Thus, there is always some doubt that they are the true deformation history.

Using the various relationships derived from combinatorial mathematics levels of confidence can be placed on deformation histories. Small numbers of unobserved relationships significantly reduce the confidence levels that may be placed on deformation histories. Younging tables (Fig. 7) provide a systematic, repeatable and rapid method of constructing deformation histories.

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